The Monte Carlo Method to analysis the difference amplifier considering the components tolerance

<u>ROCHA, F. S[1]</u>, H. O. MOTA[2], MAIA, C. A.[3], VASCONCELOS, F. H.[4]

[1] Electrical Engineering Graduate Program; [2,3,4] Department of Electrical Engineering

fabianosr@ufmg.br, hilton@cpdee.ufmg.br, c.amaia04@gmail.com, fvasc58@gmail.com

Abstract: Operational amplifier is a special type of electronic device that along other external components plays an important role in circuit design to implement a number of mathematical operations. Their present is of undeniable importance for the low-level analog signals conditioning like those employed in measurement where accuracy is high demand. However, it should be considered that electronics devices such as resistors and even OpAmps are not ideal can may introduce unexpected errors in measurements. In this work, the effect of component tolerance and its statistical characteristic will be analyzed using the Monte Carlo simulation method.

Keywords: Electronic Circuit, Monte-Carlo Method, Uncertainty.

1. INTRODUCTION

In real world, a number of assumptions made during the phase of discrete and integrated electronic circuits design lead to results different from those initially expected. Ordinary modeling stage lead the designer to somehow minimize the complexities involved and take the elements by its nominal values. The Operational Amplifiers (OpAmps) are taken as ideal devices [1].

Real devices cannot be perfectly manufactured. Even if all adjustments are made and production procedures kept constants, variability among units can still be observed. The uncertainty of electronic component is also related to the inherent uncertainties of the semiconductor manufacturing process, for example, imperfections along the etching steps or even misalignment between photomasks [2].

Manufacturers, aware of this situation, classify components according to tolerances, that is, an allowable amount of variation of the specified quantity, particularly in the dimensions or nominal value of components, bound by lower and upper limits [2].

It's well known that for devices such as resistors the production process will result in a normal (Gaussian) distribution, unless the manufacturing machine is defective [2] [3].

An electronic circuit output can be modeled through a measurement model which relate the measurand to the nominal values (inputs) of electronic components that integrate the circuit. In a probabilistic approach, each input quantity, X_i, is firstly described as Probability Density Function (PDF) [4] [5].

The Law of Propagation of Uncertainty (LPU) assumes a Gaussian output, which can significantly change the coverage interval estimated when measurand is non-gaussian [5]. On the other hand, the Monte Carlo Method (MCM) is a computational algorithm that iteratively provides the measurand PDF and has been widely applicated in electronic designs industry and investigation studies [7]. The measurement result is directly obtained by PDF statistics and its sufficiently consistent as to analytical method [4]. Furthermore, the MCM allows more realistic results since non-closed-form PDFs can be propagated through the measurement model, $Y = f(X_i)$, such as electronic components values.

Having in mind this information, the purpose of this work is to analyze how the randomness in the quantity value of a component, mainly resistors, can affect the behavior of difference amplifier circuit.

The effect on difference amplifier caused by resistors imbalance has been systematically investigated by introducing a deterministic imbalance factor, that represents the most deviation caused by one or more resistors in the circuit [1], however, assuming the electronic components variability only by their extreme values [6] or an inconsistent PDF will result in an estimate more pessimistic than in fact could be.

This paper is organized in three sections. Following this introduction, section II describers how the input quantities PDFs can affect the measurement result in a simple example. Section III, shows the simulation results and examine the figure merit of difference amplifier.

2. EQUIVALENT PARALLEL RESISTANCE

Realistic random processes and physics phenomena can be simplified by approximate probabilistic models. For example, the nominal value of an electronic device can be associated with a probability, p. In this case, the random variable, X_i, consists of the nominal value of each unit, x_k, that is produced by manufacturer. If no other information is given, the most simplified statistical model leads one to assume that nominal value of each electronic component produced, is equally likely to be assigned to the quantity within a value limit (rectangular distribution), regard that manufacturing process is was kept under statistical control.

Conveniently, a simple example is be used to illustrate how randomness can affect the behavior of a parallel circuit that will be modeled by the, well known, following expression:

$$R^{-1} = [R_1(1 \pm \Delta R_1)]^{-1} + [R_2(1 \pm \Delta R_2)]^{-1} + \ldots [R_N(1 \pm \Delta R_N)]^{-1} \quad (1)$$

The output quantity is directly related to the inputs, R_N , and tolerance, ΔR_N , through a functional relationship (1), where N denotes N-th resistor of the model. Once the nominal value and the associated uncertainties for each input quantity can be taken from the manufacture's specification or other previously available information (Type-B evaluation), the result would be as reliable as whether obtained by repeated measurements (Type-A evaluation) [4].

Manufactured resistors taken from a mass production process have specified tolerances. The nominal value of the resistor can be statistically represented by a Gaussian PDF [3] as shown in figure 1 (full line). However, along production, the manufacturer selects by measurement, the devices from a batch and them grade them in different range values or tolerances, as shown by figure 1 (hatched) [2] [3].

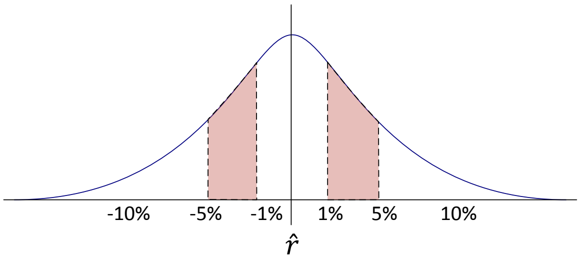

Figure 1. PDF of batch (full line) and 5 % resistor values of a single grade (hatched).

By MCM the measurand discrete representation is obtained numerically, and its statistics can be computed if discrete representation is to be organized in a non-decreasing order, a probability distribution, G_R, will be obtained [5].

A parallel circuit example is investigated using N=3, 6, 9 and 18 resistors to compare the endpoints of 95 % coverage intervals and standard deviation over a Monte Carlo simulation considering $\Delta R_{1,2\ldots N} = 0.10$ and estimative, $\hat{r}_{1,2\ldots N} = 1\,\Omega$. Figure 2 shows the multimodal PDF for R as result of input bimodal PDF, $R_{1,2,3} \in \mathbb{R}^M$, propagated by (1), where M is the number of Monte Carlo trials.

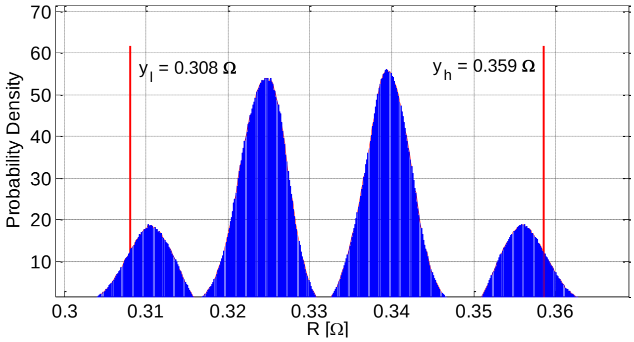

Figure 2. PDF for the R of the three resistors in paralell where $R_1 = R_2 = R_3 = 1.00 \pm 0.10\,\Omega$ using $M = 5 \times 10^6$. The continuous vertical red lines denote the low and high endpoints, y_l, y_h, of the 95 % interval.

The final result of the measurement is $\hat{r} = 0.332\,\Omega$ with a standard uncertainty $u(r) = 0.014\,\Omega$ and endpoints corresponding the 95 % coverage interval is [0.308, 0.359] Ω.

Table 1 summarizes the MCM for N=3, 6, 9 and 18 resistors.

Table 1. Results for 10 % resistors in parallel evaluated using Monte Carlo Method.

N	$\hat{r}$ /Ω	$u(r)$ /Ω	$u_{95\%}(r)$/%	Shortest 95 % coverage interval /Ω
3	0.332	0.0140	7.68	[0.308, 0.359]
6	0.166	0.0048	5.35	[0.157, 0.175]
9	0.111	0.0026	4.45	[0.106, 0.116]
18	0.055	0.0010	3.26	[0.054, 0.057]

3. DIFFERENCE AMPLIFIER MODELING

The difference amplifier is a special type of amplifier which is widely used in industry, instrumentation and measurement of noisily quantities, where precision and gain accuracy are essential. This topology fundamentally performs the difference or subtraction between two inputs V_2 and V_1 linearly proportional to gain, A_d. The difference amplifier output can be defined by two components of its input: common-mode and differential-mode given by equation 2 [1].

$$V_o = A_d V_d + A_c V_c \tag{2}$$

Where:

A_d – differential-mode gain;

V_d – differential voltage $(V_2 - V_1)$;

A_c – common-mode gain;

V_c – common-voltage $(V_1 + V_2)/2$

Differential amplifiers must be increases differential low-intensity signals and then rejects common voltage (grounded reference) signals between its input terminals. However, in most applications, the instrument reference is not completely isolated from source ground. Hence, the output includes an unwanted voltage which is amplified along circuit producing errors in the output [1].

A difference amplifier is typically designed with four resistors as shown figure 3.

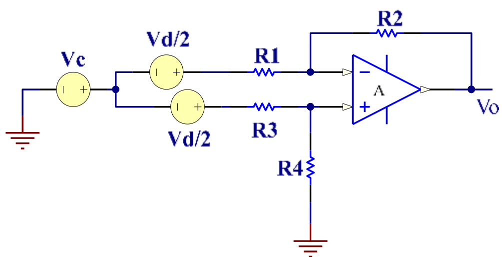

Figure 3. Difference amplifier designed by general-purpose OpAmp.

The difference amplifier common voltage rejection can be defined in terms of the figure of merit denominated common-mode rejection ratio (CMRR) given by equation 3.

$$CMRR = 20\log_{10}\left|{}^{A_d}/_{A_c}\right| \quad (3)$$

The drawback for this implementation, as figure 3, is high sensibility of CMRR regarding the resistors values, providing worst common-mode rejection. Considering an ideal amplifier, the CMRR value can be modeled by equation 4 in terms of the resistances that compose the circuit shown by figure 3.

$$CMRR = 20\log_{10}\left|\frac{R1R4+R2R3+2R2R4}{2(R1R4-R2R3)}\right| \quad (4)$$

The estimative and uncertainty associated to *CMRR* give by MCM considering the bimodal PDF is shown by table 2. For this purpose, consider the unity gain with estimative $\hat{r}_1 = \hat{r}_2 = \hat{r}_3 = \hat{r}_4 = 380$ kΩ, with ΔR_N of tolerance.

Table 2. Results for common-mode rejection ratio of the difference amplifier evaluated using the MCM.

ΔR_N/%	$\widehat{cmrr}$ /dB	$u(cmrr)$ /dB	Shortest 95 % coverage interval /Ω
10	29.86	11.81	[15.30, 52.87]
5	35.65	9.74	[22.01, 55.15]
1	49.63	9.59	[35.91, 68.74]
0.1	69.61	9.61	[55.90, 88.75]

Figure 4 and figure 5 shows, respectively, the PDF and distribution function for CMRR of difference amplifier corresponding 5 % resistors implementation.

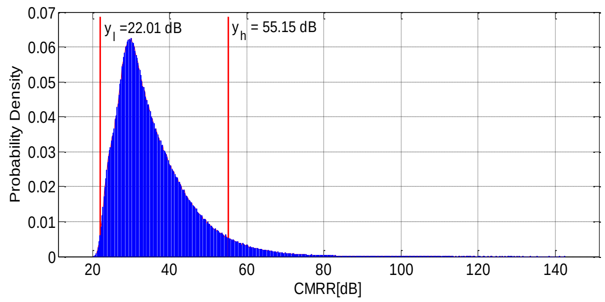

Figure 4. PDF of the CMRR considering 5 % tolerance resistors.

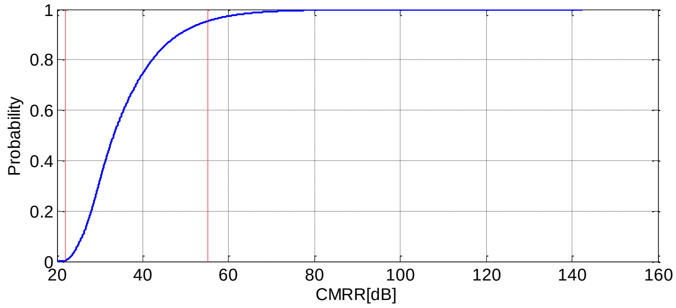

Figure 5. CDF of the CMRR considering 5 % tolerance resistors.

4. CONCLUSION

In the statistical approach, the values of the components devices such as resistors must be modeling by own probability distributions considering the range of tolerance and classification method.

5. REFERENCES

[1] FRANCO, Sergio. Design with operational amplifiers and analog integrated circuits. McGraw-Hill series in electrical and computer engineering, 2015.

[2] SPENCE, Robert; SOIN, Randeep Singh. Tolerance design of electronic circuits. World Scientific, 1997.

[3] COX, M. G.; HARRIS, P. M. Software support for metrology best practice guide no. 6: uncertainty evaluation. NPL Report, 2006.

[4] JCGM 100:2008 – Evaluation of measurement data – Guide to the expression of uncertainty in measurement. Joint Committee for Guides in Metrology; 2008.

[5] JCGM 101:2008 – Evaluation of measurement data - Supplement 1 to the “Guide to the expression of uncertainty in measurement” – Propagation of distributions using a Monte Carlo method. Joint Committee for Guides in Metrology; 2008.

[6] SMITH, Walter M. Worst case circuit analysis-an overview (electronic parts/circuits tolerance analysis). In: Reliability and Maintainability Symposium, 1996 Proceedings. International Symposium on Product Quality and Integrity., Annual. IEEE, 1996. p. 326-3.

[7] HAMANN, Jerry C. et al. Using Monte Carlo simulations to introduce tolerance design to undergraduates. IEEE Transactions on Education, v. 42, n. 1, p. 1-7, 1999.

ACKNOWLEDGEMENTS

This work has been supported by the Brazilian agency CAPES.