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Mathematical Analysis of Regression Models Applicable to the Force Transducers Classification

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Resumo: Os transdutores de força são aplicados nas mais diversas áreas da tecnologia, engenharia e metrologia. São utilizados na calibração de durômetros, e de máquinas de ensaio de tração e compressão. Os transdutores de força também são constituintes das balanças eletrônicas utilizadas diariamente nas mais diversas atividades econômicas, científicas e industriais. A classificação de um transdutor de força é de importância primordial para a determinação da sua aplicabilidade. A inexistência de uma definição de uma metodologia para a determinação do modelo de regressão é um empecilho para que se tenha uniformidade no cálculo do erro de interpolação relativo. Este trabalho mostra que a ausência deste tipo de normalização acarreta possíveis classificações diferentes dos transdutores de força dependendo da metodologia de regressão utilizada.

Palavras-chave: transdutores de força; erro de interpolação relativo; metodologia de regressão polinomial; classificação de transdutores de força.

Abstract: The force transducers are applied in the most diverse areas of technology, engineering and metrology. They are used in calibration of durometers as well as tension and compression testing machines. The force transducers are also constituents of electronic weighing-machines used daily in many economic, scientific, and industrial activities. The classification of a force transducer is of paramount importance in determining its applicability. The lack of a definition for a methodology to the determination of the regression model is an obstacle for one reaching some uniformity in the calculation of the relative interpolation error. This work shows that the absence of this type of normalization leads to possible different classifications of the force transducers depending on the regression methodology used.

Keywords: force transducers; relative interpolation error; polynomial regression methodology; force transducers´ classification.

1. INTRODUCTION

Among the statistical methods that provide practical application to metrology is the use of polynomial regression models. This type of tool is applied when one wishes to study the behavior of two or more variables, that is, how a response varies as a function of an input variable. In order to establish a polynomial regression model (equation) between two variables, the first step is collecting a sort of observations (x_i, y_i) that will constitute the points of the dispersion plot [1]. This process is called regression and is based on an adjustment method called the Least Squares

Method (LSM). This methodology consists in adjusting the set of data to a function that minimizes the experimental variance of the set [1-2].

The force transducer is a transfer standard of the quantity force that is suitably traced to Inmetro´s primary force machine. This calibration consists in performing several loading cycles, where the force generated by the primary force machine is compared with the electrical signal in mV/V produced by the force transducer. This signal is displayed on a digital indicator that is connected to the force transducer during the calibration process. Table 1 shows the force transducers classification taking into account the relative interpolation error as of ISO 376 [3] since it is generally the main classification criterion.

Table 1 - Classification of force transducers according to ISO 376.

Class	f_c (%)
00	± 0.025
0.5	± 0.05
1	± 0.10
2	± 0.20

ISO 376 standard establishes the methodology of calibration and classification of force transducers. However, for the relative interpolation error (f_c) that is a criterion for the force transducer classification there is no definition on the type of 3rd degree polynomial to be determined and applied to correlate force and potential difference values (mV/V) observed during the calibration process in a force transducer [3].

Analysis of two approaches of 3rd degree polynomial regression and their impacts on the metrological force transducers classification was the objective of this paper.

2. MATERIALS AND METHODS

For the accomplishment of this work data from ISO 376 calibration of two HBM force transducers (TF01 and TF02) realized by two national metrology institutes were analyzed. TF01 has Inmetro´s calibration certificate DIMCI 1113/2013 whereas TF02 has LNE´s certificate L120020/10. Inmetro means National Institute of Metrology, Quality and Technology, from Brazil. LNE means National Laboratory of Metrology and Testings, from France. TF01 data: nominal load of 3000 kN, C_3 model, and serial number 89100. TF02 data: nominal load of 10 N, U_1 model, and serial number E28522.

Regression analysis for the interpolation model determination applied on TF01 and TF02 consisted of two approaches for establishing two third degree polynomial models. The first one looks for finding out a polynomial where all coefficients are associated with the independent variable, according to equation (1).

$$y = a \cdot x^3 + b \cdot x^2 + c \cdot x \quad (1)$$

The second approach provides a polynomial that also includes the portion that is not associated with the independent variable, according to equation (2).

$$y = a \cdot x^3 + b \cdot x^2 + c \cdot x + d \quad (2)$$

By using LSM it´s possible to estimate the coefficients of the regression model so that the sum of the squares of each deviation (d_i) from the experimental values is expressed by equations (3) and (4).

$$d_i = y_i - \hat{y}_i \quad (3)$$

$$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (4)$$

The value of $\sum_{i=1}^{n} d_i^2$ will reach a minimum when its partial derivatives are estimated with respect to the appropriate coefficients and this calculus has to be applied to each of the two regression proposed models as shown by equations (5) to (8).

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial a} = 0 \quad (5)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial b} = 0 \quad (6)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial c} = 0 \quad (7)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial d} = 0 \quad (8)$$

Thus, the LSM determination of the regression model regarding to equation (1) is calculated by solving the system of equations formed by equations (9), (10), and (11).

$$a\sum x_i^6 + b\sum x_i^5 + c\sum x_i^4 = \sum x_i^3 y_i \quad (9)$$

$$a\sum x_i^5 + b\sum x_i^4 + c\sum x_i^3 = \sum x_i^2 y_i \quad (10)$$

$$a\sum x_i^4 + b\sum x_i^3 + c\sum x_i^2 = \sum x_i y_i \quad (11)$$

To determine the regression model established by equation (2) it is necessary solving the system of equations represented by equations (12), (13), (14), and (15).

$$a\sum x_i^6 + b\sum x_i^5 + c\sum x_i^4 + d\sum x_i^3 = \sum x_i^3 y_i \quad (12)$$

$$a\sum x_i^5 + b\sum x_i^4 + c\sum x_i^3 + d\sum x_i^2 = \sum x_i^2 y_i \quad (13)$$

$$a\sum x_i^4 + b\sum x_i^3 + c\sum x_i^2 + d\sum x_i = \sum x_i y_i \quad (14)$$

$$a\sum x_i^3 + b\sum x_i^2 + c\sum x_i + n\cdot d = \sum y_i \quad (15)$$

3. RESULTS AND DISCUSSION

The calibration results that were used for the determination of f_c for both TF01 and TF02 are shown in Tables 2 and 3, respectively.

Table 2 - TF01 calibration data.

Applied Force (kN)	X_1 (mV/V)	X_3 (mV/V)	X_5 (mV/V)	$\bar{X}_r$ (mV/V)
300	0.19996	0.19998	0.19998	0.19997
600	0.39992	0.39992	0.39992	0.39992
900	0.60013	0.60009	0.6001	0.60011
1200	0.80042	0.80033	0.80035	0.80037
1500	1.00069	1.00056	1.00055	1.00060
1800	1.20104	1.20086	1.20081	1.20090
2100	1.40127	1.40106	1.40099	1.40111
2400	1.60139	1.60116	1.60104	1.60120
2700	1.80152	1.80124	1.8011	1.80129
3000	2.00149	2.00121	2.00102	2.00124

Calibration scheme comprised increasing nominal loads e.g. 10%, 20%, and so on.

Equations (16) and (17) [3] show, respectively, f_c, the mean deformation value of a non-rotating transducer, and $\bar{X}_r$, the mean value of deformations with rotation.

Table 3 - TF02 calibration data.

Applied Force (N)	X_1 (mV/V)	X_3 (mV/V)	X_5 (mV/V)	$\bar{X}_r$ (mV/V)
1	0.19776	0.19798	0.19782	0.19785
2	0.39901	0.39928	0.39909	0.39913
3	0.59947	0.59978	0.59964	0.59963
4	0.79981	0.80015	0.79997	0.79998
5	1.0001	1.00042	1.0003	1.00027
6	1.20034	1.20069	1.2006	1.20054
7	1.40062	1.40098	1.40085	1.40082
8	1.60095	1.60125	1.60118	1.60113
9	1.80124	1.8015	1.80161	1.80145
10	2.00152	2.00169	2.00189	2.00170

$$\bar{X}_r = \frac{X_1 + X_3 + X_5}{3} \quad (16)$$

$$f_c = \frac{\bar{X}_r - X_a}{X_a} \times 100 \quad (17)$$

where: X_1 - value of the first series of deformation of a transducer for a given force; X_3 - value of the second series of deformation of a transducer for a given force; X_5 - value of the third series of deformation of a transducer for a given force; X_a - interpolation value of the deformation from the regression model.

For TF01 the two 3[rd] degree regression models obtained by LSM for both Inmetro and LNE are presented in equations (18) and (19), respectively. Similarly, for TF02, equations (20) and (21) are shown below.

$$y = -2.08011 \times 10^{-13} \cdot x^3 + 9.3799 \times 10^{-10} \cdot x^2 + 0.000666132 \cdot x \quad (18)$$

$$y = -2.20606 \times 10^{-13} \cdot x^3 + 1.006 \times 10^{-10} \cdot x^2 + 0.000666024 \cdot x + 4.61111 \times 10^{-5} \quad (19)$$

$$y = -1.46864 \times 10^{-5} \cdot x^3 + 0.00024812 \cdot x^2 + 0.199131941 \cdot x \quad (20)$$

$$y = 9.20163 \times 10^{-6} \cdot x^3 - 0.000181865 \cdot x^2 + 0.201401307 \cdot x - 0.003239222 \quad (21)$$

The regression model defined by equations (18) and (20) applied to TF01 and TF 02 is the methodology used by Inmetro (and also by Germany´s PTB) while equations (19) and (21) for TF02 are used by LNE. The physical phenomenon of the force transducer calibration represented by Inmetro's method seems to be better since when there is no force applied to the force transducer no deformation occurs in it.

Table 4 and 5 show for TF01 and TF02 the calculation of X_a and f_c using the regression model presented in equations (18), (19), (20), and (21), respectively. Moreover, these tables show the relative interpolation error difference (%) from the two regression models for TF01 and TF02, respectively, taking into consideration Inmetro´s values are the reference ones because their better behavior as physical phenomena compared to LNE´s ones.

Table 4 - Calculation of the deformation value and relative interpolation error due to two regression models for TF01.

$X_{a\,(18)}$ (mV/V)	$f_{c\,(18)}$ (%)	$X_{a\,(19)}$ (mV/V)	$f_{c\,(19)}$ (%)	**Relative interpolation error difference (%)**
0.19992	0.027	0.19994	0.018	-35.64
0.39997	-0.013	0.39998	-0.014	6.29
0.60013	-0.003	0.60012	-0.003	-24.31
0.80035	0.002	0.80034	0.003	40.40
1.00061	-0.001	1.00060	0.000	-78.56
1.20086	0.003	1.20086	0.003	2.04
1.40109	0.001	1.40109	0.001	-16.64
1.60124	-0.003	1.60125	-0.003	11.27
1.80130	-0.001	1.80130	-0.001	25.07
2.00122	0.001	2.00122	0.001	26.03

Table 5 - Calculation of the deformation value and relative interpolation error due to two regression models for TF02.

$X_{a\,(20)}$ (mV/V)	$f_{c\,(20)}$ (%)	$X_{a\,(21)}$ (mV/V)	$f_{c\,(21)}$ (%)	**Relative interpolation error difference (%)**
0.19937	-0.758	0.19799	-0.069	-90.94
0.39914	-0.003	0.39891	0.054	1680.00
0.59923	0.066	0.59958	0.009	-86.52
0.79956	0.052	0.80005	-0.009	-83.68
1.00003	0.025	1.00037	-0.010	-60.42
1.20055	-0.001	1.20061	-0.005	684.40
1.40104	-0.016	1.40081	0.000	-99.14
1.60142	-0.018	1.60104	0.005	-71.10
1.80158	-0.007	1.80135	0.006	-21.96
2.00145	0.013	2.00179	-0.004	-65.10

According to Table 1 and analyzing Table 4 and 5, the regression model applied by equation (18) and (20) classified TF01 and TF02 as class 0.5 and 2, respectively. Following the same reasoning, the model developed by equations (19) and (21) showed TF01 and TF02 as class 00 and 1, respectively. So, force transducer classification depends on the regression methodology used. As a consequence, some guiding on the methodology used to obtain the 3^{rd} degree polynomial should be considered in a future ISO 376 revision. This way an international metrology level uniformity on determination of the relative interpolation error could result.

As a suggestion interlaboratory comparisons and proficiency tests involving force determination should define the methodology for calculating the relative interpolation error.

4. CONCLUSIONS

- Regression models based on polynomials of 3^{rd} degree shouldn´t use the independent variable approach since more reliable results can be obtained in calibrating force transducers according to ISO 376.

- The interpolation method determines the 3^{rd} degree polynomial that provides the relative interpolation error which directly impacts the force transducers classification.

- Future ISO 376 revisions should define the better methodology for determining the 3^{rd} degree polynomial used to calculate the relative interpolation error in force transducer calibration.

5. REFERENCES

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