Metrologia
Fortaleza - CE 2017

Mathematical Comparison between Excel and Least Square Method Applicable to Force Transducers Classification

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Resumo: Os transdutores de força são usados na engenharia para garantir a confiabilidade metrológica da grandeza Força aplicada cotidianamente em diversas atividades econômicas, científicas e industriais. Portanto, é necessário analisar criticamente o resultado de calibração obtido através de um determinado software, por mais maduro e de uso generalizado ele seja. O processo de validação é essencial para encontrar desvios ou erros no processo de determinação do resultado de uma medição. Neste estudo de caso, mostra-se que o método gráfico de regressão usado pelo software Excel não é o do Método de Mínimos Quadrados. Esta diferença ocasiona mudanças inesperadas na classificação dos transdutores de força devido à sua relação com o erro relativo de interpolação.

Palavras-chave: Excel; Método de Mínimos Quadrados; Erro relativo de interpolação; Classificação de transdutores de força.

Abstract: Force transducers have been used in engineering in order to assure the metrological reliability of the quantity Force applied daily in many economic, scientific and industrial activities. Therefore, it is necessary analyzing critically the calibration result obtained through any software, no matter it is a mature and of common use one. The validation process is essential to find out deviations or errors in the process of obtaining the result of any measurement. In this case study, it´s been shown the graphical regression method used by Excel software is not the Least Square Method one. This difference causes some not expected changes in the classification of force transducers due to its relationship with the relative interpolation error.

Keywords: Excel; Least Square Method; Relative Interpolation Error; Force Transducer Classification

1. INTRODUCTION

Development of new materials, products, etc depends on reliable measurements which mean measurements with both metrological traceability and reliability. This way goods are considered suitable for use, processing and analysis. It is often necessary to establish some mathematical relationship between variables present in a given process [1]. E.g. the mathematical model has to be able of representing the phenomenon being analyzed, controlled or studied. The mathematical model is considered satisfactory when the difference between experimental data and theoretical ones coming from a mathematical model tends to a minimum value, i.e. being able to tend to zero. For this "checking" the Least Square Method (LSM) [1], [2] should be used.

A force transducer is a metrological transfer standard of the quantity Force that is suitably traced to Inmetro´s primary force machine (PFM). Calibration consists in performing several loading cycles, where the force generated by PMF is compared with the electrical signal in mV/V produced by the force transducer. This signal is displayed on a digital indicator that is connected to the force transducer during the calibration process. A force transducer can be classified according to the differences between theoretical and experimental values of deformation. This difference gives rise to the relative interpolation error [3]. Table 1 shows the force transducers classification taking into account the relative interpolation error $f_c(\%)$ as of ISO 376 [3] since it is generally the main classification criterion.

Table 1 - Classification of force transducers according to ISO 376.

Class	$f_c(\%)$
00	± 0.025
0.5	± 0.05
1	± 0.10
2	± 0.20

2. MATERIALS AND METHODS

For the present work, three different HBM force transducers were used. The force transducers were identified as FT01 (model Z4 and 500 kN of load capacity), FT02 (model C3 and 3000 kN of load capacity), and FT03 (model Z30 and 1 kN of load capacity). An analysis consists of achieving graphically the 3rd degree polynomial model through Excel and comparing it with the polynomial determined by the LSM.

The regression analysis process for the determination of the interpolation model applied consisted in a polynomial such that all coefficients are associated with the independent variable, according to equation (1).

$$y = a \cdot x^3 + b \cdot x^2 + c \cdot x \quad (1)$$

By using LSM it is possible estimating the coefficients of the regression model so that the sum of the squares of the deviations (d_i^2) from the experimental values be the smallest as possible, according to equations (2) and (3).

$$d_i = y_i - \hat{y}_i \quad (2)$$

$$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (3)$$

The value of $\sum_{i=1}^{n} d_i^2$ will assume a minimum when the partial derivatives with respect to the coefficients follow each of the regression models proposed by equations (4) to (7).

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial a} = 0 \quad (4)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial b} = 0 \quad (5)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial c} = 0 \quad (6)$$

$$\frac{\partial \sum_{i=1}^{n} d_i^2}{\partial d} = 0 \quad (7)$$

Thus, the LSM determination of the regression model regarding to equation (1) is calculated by solving the system of equations formed by equations (8), (9), and (10).

$$a \sum x_i^6 + b \sum x_i^5 + c \sum x_i^4 = \sum x_i^3 y_i \quad (8)$$

$$a \sum x_i^5 + b \sum x_i^4 + c \sum x_i^3 = \sum x_i^2 y_i \quad (9)$$

$$a \sum x_i^4 + b \sum x_i^3 + c \sum x_i^2 = \sum x_i y_i \quad (10)$$

A matrix calculation that takes into account the matrices *T*, *A*, *B*, and *C* (equations (11), (12), (13) and (14), respectively) below is the key for solving the system of equations (8), (9), and (10).

$$[T] = \begin{bmatrix} \sum x_i^6 & \sum x_i^5 & \sum x_i^4 \\ \sum x_i^5 & \sum x_i^4 & \sum x_i^3 \\ \sum x_i^4 & \sum x_i^3 & \sum x_i^2 \end{bmatrix} \quad (11)$$

$$[A] = \begin{bmatrix} \sum x_i^3 y_i & \sum x_i^5 & \sum x_i^4 \\ \sum x_i^2 y_i & \sum x_i^4 & \sum x_i^3 \\ \sum x_i y_i & \sum x_i^3 & \sum x_i^2 \end{bmatrix} \quad (12)$$

$$[B] = \begin{bmatrix} \sum x_i^6 & \sum x_i^3 y_i & \sum x_i^4 \\ \sum x_i^5 & \sum x_i^2 y_i & \sum x_i^3 \\ \sum x_i^4 & \sum x_i y_i & \sum x_i^2 \end{bmatrix} \quad (13)$$

$$[C] = \begin{bmatrix} \sum x_i^6 & \sum x_i^5 & \sum x_i^3 y_i \\ \sum x_i^5 & \sum x_i^4 & \sum x_i^2 y_i \\ \sum x_i^4 & \sum x_i^3 & \sum x_i y_i \end{bmatrix} \quad (14)$$

According to the equations (15) to (17) below the coefficients of equation (1) are determined by the following operations among the determinants of matrices *T*, *A*, *B*, and *C*.

$$a = \frac{\det(A)}{\det(T)} \quad (15) \qquad b = \frac{\det(B)}{\det(T)} \quad (16)$$

$$c = \frac{\det(C)}{\det(T)} \quad (17)$$

3rd polynomial estimate with Excel 2007 is made by using the trend line adjustment function. Thus, one selects both the function has to pass through the origin (0,0) of the Cartesian axes and the adjustment is made by a polynomial of the third degree.

3. RESULTS AND DISCUSSION

3.1 Determination of the polynomials from the 3rd degree for FT01, FT02, and FT03

Transducers TF01, TF02, and TF03 were calibrated and the resulting data were analyzed in order to determine the third degree polynomial, according to equation (1). The third degree polynomial in the case of force transducers requires the intercept of the model adjusted be located at the point (0,0) which characterizes the best representation of the physical phenomenon. This model represents that when there is no force there will be no deformation registered by the force transducer.

The determination of the 3rd degree polynomial is crucial for the calculation of the relative interpolation error "f_c" which is determined from both equations (18) and (19):

$$\bar{X}_r = \frac{X_1 + X_3 + X_5}{3} \quad (18)$$

$$f_c = \frac{\bar{X}_r - X_a}{X_a} \times 100 \quad (19)$$

where:

X_r - mean value of deflections with rotation; X_1 - value of first series of deformation of a transducer for a given force; X_3 - value of second series of deformation of a transducer for a given force; X_5 - value of third series of deformation of a transducer for a given force; X_a - interpolation value of deformation from the regression model.

According to LSM, the third degree polynomials were determined for the three force transducers analyzed. Table 2 shows the coefficients *a*, *b*, and *c* for the force transducers FT01, FT02, and FT03.

Table 2 - 3rd polynomial coefficients by LSM for FT01, FT02, and FT03.

Mathematical Coefficients by LSM	FT01	FT02	FT03
a	-7.37E-12	-2.08E-13	-1.01E-03
b	5.97E-09	9.38E-10	1.92E-03
c	-4.00E-03	6.66E-04	2.00 E+00

Table 3 shows the coefficients of the 3rd degree polynomial coming from the calibration data and Excel 2007 graphical adjustment. These data will be important for determining the relative interpolation error and then the metrological force transducers classification according to ISO 376 standard. Comparing data of both Tables 2 and 3, it can be noted coefficient *c* of the 3rd degree polynomial has the same value regardless of the method used for its determination. However, coefficients *a* and *b* calculated by

Excel present higher values by comparing with the values obtained from LSM.

Table 3 - 3rd polynomial coefficients provided by Excel for FT01, FT02, and FT03.

Mathematical Coefficients by Excel graphical adjustment	FT01	FT02	FT03
a	-5.00E-11	-3.12E-13	-6.92E-03
b	6.00E-09	1.12E-09	5.43E-03
c	-4.00E-03	6.66E-04	2.00E+00

3.2 Calculation of Relative Interpolation Error

Relative interpolation errors were calculated for the three force transducers studied by using the 3rd degree polynomials determined by the Least Squares Method in Table 4 and by the Excel in Table 5. Tables 4 and 5 contain the maximum values of relative interpolation error (f_c) and transducer classification according to ISO 376.

Table 4 - Maximum relative interpolation error and force transducer classification by LSM for FT01, FT02 and FT03.

Force Transducer	FT01	FT02	FT03
Maximum f_c - LSM	0.016	0.027	0.005
Force Transducer Classification	00	0.5	00

Table 5 - Maximum relative interpolation error and force transducer classification by Excel for FT01, FT02 and FT03.

Force Transducer	FT01	FT02	FT03
Maximum f_c - Excel	0.19	0.072	0.153
Force Transducer Classification	2	1	2

From the analysis of Tables 4 and 5 it can be realized that the force transducers classification varies somewhat depending on which the method of determination of the polynomial of the third degree was used. Excel 2007 presents some mistake in its graphical determination of the polynomial coefficients.

Thus, when using the graphic method to determine the polynomial, it is possible to degrade the metrological quality of the calibrated force transducer. Therefore a validation procedure for comparing both methodologies has to be applied.

4. CONCLUSIONS

- The methodology for determination of the 3rd degree polynomials with interception in the origin of the Cartesian axes presented unexpected results when the polynomials coefficients were obtained from the Excel software.

- On classifying force transducers according to ISO 376, there is some evidence that Excel's graphical methodology tends to depreciate the metrological quality of the force transducers.

- The validation is important for the suitable process that is desired for classifying and/or calculate parameters of a given process. Therefore, in the area of classification of transducers by the interpolation error, the application of least squares methodology was fundamental to detect that there was an error in the graphical calculation of Excel software.

- This work highlights that Excel and other calculation programs users has to proceed to data validation. This way metrological assurance and reliability of the results of a given measurement will result.

5. REFERENCES

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