

## Determinação da incerteza de medição para o ensaio de integral J através do método de Monte Carlo

## Determination of the measurement uncertainty for the J-integral test using the Monte Carlo method

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**Resumo:** O GUM sugere um método de análise de incerteza de medição através de modelos linearizados, que não se adequam a modelos não lineares, como para o ensaio de integral J. O método de Monte Carlo é uma aproximação numérica que realiza uma amostragem aleatória, utilizando dados fornecidos pelo usuário, evitando este problema. A incerteza de medição para o ensaio de integral J foi calculada utilizando este método e as equações apresentadas pela norma ASTM E1820:2016 para corpos de prova do tipo SE(B).

**Palavras-chave:** Incerteza de medição; Método de Monte Carlo; Integral J.

**Abstract:** The GUM suggests a method for analyzing the measurement uncertainty by linearized models, which is not adequate to the non-linear ones, as the J-integral test. Monte Carlo method is a numerical approach that performs random sampling from probability distribution using inputs provided by the user, which avoids this problem. The measurement uncertainty of the J-integral was calculated using this method and the equations provided by the ASTM E1820:2016 standard for SE(B) shaped specimens.

**Keywords:** Measurement uncertainty; Monte Carlo method; J-integral.

### 1. INTRODUCTION

The Guide to the Expression of Uncertainty in Measurement (GUM) suggests a process for the estimation of measurement uncertainty of linearized models, by interrelating input quantities in relation to one output quantity, normally associated with a Gaussian distribution

or a t-distribution. For models where linearization provides an inadequate representation or the probability density distribution for the output that differs from the distributions above, the GUM proposes the use of a propagation of distributions using a Monte Carlo method. This method is a numerical approach, which provides a numerical approximation to the distribution function of the

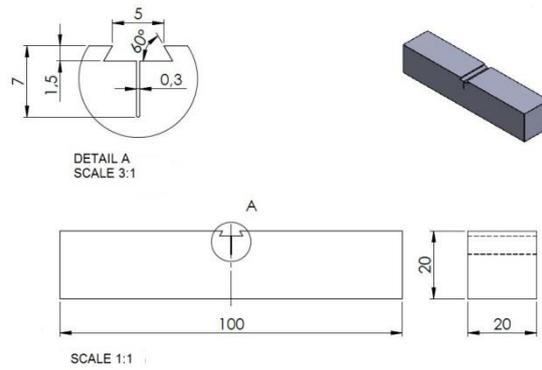
output quantity, performing random sampling from probability distributions [1-3].

J-integral is a contour integral that analyzes the fracture toughness of material, whose elastic-plastic deformation is non-linear. It is used to characterize the local stress-strain field around the crack front, by enclosing the crack front from one crack surface to the other. It analyzes the mechanical fields near crack tips in both linear elastic and non-linear elastic materials. The difficulty in measuring the uncertainty of this method is due to its non-linear distribution and the complication to calculate the sensitivity coefficients and effective degrees that turns the GUM uncertainty framework a non-practical and time-consuming method [4,5].

Thus, this paper presents a model for determining the uncertainty of the J-integral, aiming the construction of a J-R curve, based on the test method given on the ASTM E1820:2016 standard, and an example of implementation [5].

## 2. MATERIALS AND METHODS

The measurement uncertainty calculus was performed using the Monte Carlo method, according to the GUM's Supplement 1. The number of iterations chosen was 1,000,000, with a probability of coverage of 95%, which were performed using the Crystal Ball<sup>®</sup> software, a supplement of Microsoft Excel<sup>®</sup>. The specimen's shape chosen to calculate the J-integral was the single edge bend [SE(B)] illustrated on Figure 1, the standard bend specimen for three-point-bend loading.



**Figure 1.** Example of a SE(B) specimen in [mm].

The equations (1 to 4) illustrate the calculation of J-integral by the Basic Test Method, for single edge specimens, provided by the ASTM E1820:2016 [5].

$$J = J_{el} + J_{pl} \quad (1)$$

$J_{el}$  = elastic component of J

$J_{pl}$  = plastic component of J

$$J = \frac{K^2(1-\nu^2)}{E} + \frac{\eta_{pl}A_{pl}}{B_N b_0} \quad (2)$$

$K$  = see (3)

$\nu$  = Poisson's modulus

$E$  = elasticity modulus

$\eta_{pl} = 3.669 - 2.199(a_0/W) + 0.437(a_0/W)^2$  using the crack mouth opening displacement record for  $A_{pl}$

$A_{pl}$  = area under force displacement record

$B_N$  = net specimen thickness ( $B_N = B$  when no sides grooves are present)

$b_0 = W - a_0$

$$K_{(i)} = \left[ \frac{P_i S}{(B B_N)^{1/2} W^{3/2}} \right] f(a_i/W) \quad (3)$$

$P_i$  = applied load

$S$  = span (distance between the specimen supports)

$f(a_i/W) = \text{see (4)}$

$$f\left(\frac{a_i}{W}\right) = \frac{3\left(\frac{a_i}{W}\right)^{1/2} \left[ 1.99 - \left(\frac{a_i}{W}\right) \left(1 - \frac{a_i}{W}\right) \left( 2.15 - 3.93 \left(\frac{a_i}{W}\right) + 2.7 \left(\frac{a_i}{W}\right)^2 \right) \right]}{2 \left(1 + 2 \frac{a_i}{W}\right) \left(1 - \frac{a_i}{W}\right)^{3/2}} \quad (4)$$

Crystal Ball<sup>®</sup> is a user-friendly and customizable Excel add-in that easily enables Monte-Carlo simulations to be performed. Thus, using Crystal Ball<sup>®</sup> the value contained in an Excel cell can represent a random variable featured by its expected value (the value of the cell) and its assumed PDF together with a given dispersion measurement. For each parameter affecting the measurand, an Excel cell is built. The measurand value is computed in another Excel cell by applying the corresponding mathematical operations with the parameters cells. The measurand cell that contains the computed value is chosen as the forecast cell and the simulation is started once the number of trials *M* (and other features) is selected [6].

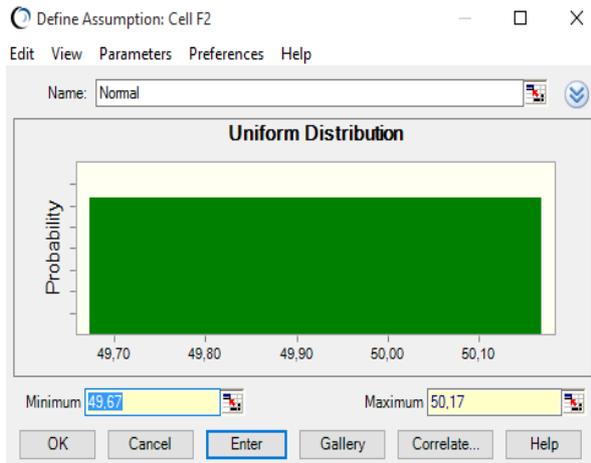
The uncertainty source of each one of the variables was considered as rectangular (uniform), supposing the worst condition possible, i.e., the most conservative situation. This is due to the objective of obtaining the measurement uncertainty values through the J-R curve, aiming the application in a small quantity of samples, being not possible the replication of the tests. The following uncertainty sources were: the acceptance criteria of the measurement instrument of *W*, the acceptance criteria of the measurement instrument of *B*, the acceptance criteria of the measurement instrument of *b<sub>0</sub>*, the acceptance criteria of the measurement instrument of *a<sub>i</sub>*, the acceptance criteria of the measurement instrument of *P<sub>i</sub>*, the acceptance criteria of the measurement instrument of *S* and the acceptance criteria of the measurement instrument of *a<sub>0</sub>*. The acceptance criteria of the measurement instruments are obtained from

standard test methods requirements. The values of *v*, *E*, *η<sub>pl</sub>* and *A<sub>pl</sub>* were considered as constants. Table 1 shows the uniform distribution length used as an input on the software for one of the test specimens [5].

**Table 1.** Standard uncertainty associated to each uncertainty source.

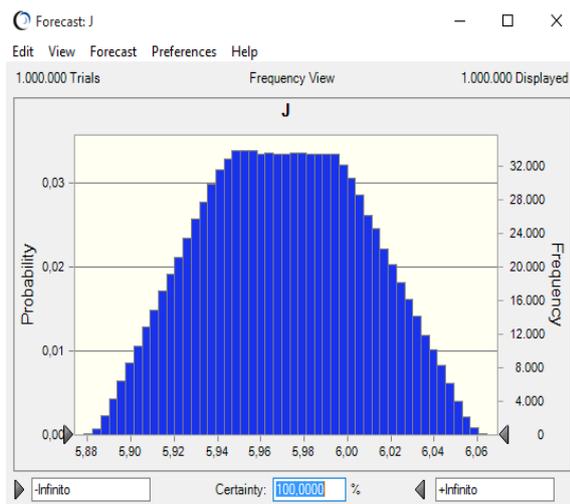
Variable	Measured value	Uniform distribution length
<i>W</i> [mm]	49.92	Max 5% of <i>W</i> = 0.2496
<i>B</i> [mm]	25.05	0.5% <i>B</i> = 0.1253
<i>b<sub>0</sub></i> [mm]	21.91	0.5% <i>b<sub>0</sub></i> = 0.1096
<i>a<sub>i</sub></i> [mm]	28.21	0.0250
<i>P<sub>i</sub></i> [N]	510.96	2% <i>P<sub>max</sub></i> = 2.3120
<i>S</i> [mm]	199.34	0.5% <i>S</i> = 0.9967
<i>v</i> (Poisson)	0.40	-
<i>Y</i> = <i>f</i> ( <i>a</i> / <i>W</i> )	1.24	-
<i>E</i> [MPa]	1000	-
<i>η<sub>pl</sub></i>	2.57	-
<i>A<sub>pl</sub></i>	1238.29	-
<i>a<sub>0</sub></i> [mm]	28.01	0.0250
<i>P<sub>max</sub></i> [N]	115.60	5% <i>P<sub>max</sub></i> = 5.7800

These data were used as input on the software to obtain the probability distribution. In addition, seven specimens were analyzed to obtain the J-R curve, which is a property of materials and gives the representativeness of the measurement uncertainty along the curve. Figure 2 shows an example of the data inputting on the software.



**Figure 2.** Defining inputs on Crystal Ball® for W variable.

After the iterations were performed, the software provided a final probability for J-integral distribution as illustrated on Figure 3.



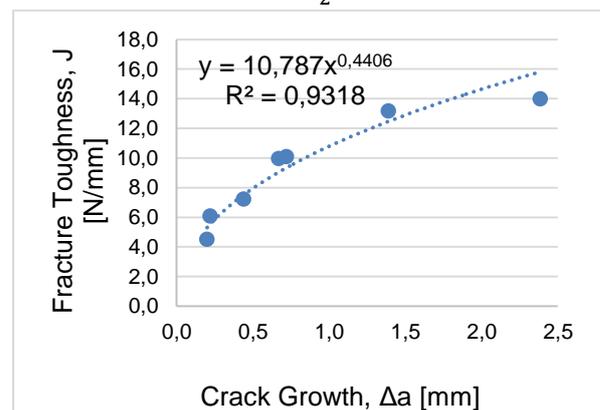
**Figure 3.** Probability distribution of J integral for one specimen.

### 3. RESULTS AND DISCUSSION

The results of the iterations provided by the software presented an estimation of the probability with its respective standard

deviations, which were used to calculate the J-integral results for each specimen and its respective expanded uncertainty. The mean value was the one provided directly from the equation (1) for each specimen and the software calculated the expanded uncertainty by relating values representing the percentiles equivalent to  $(100\% - \alpha)/2$  and to  $(100\% + \alpha)/2$ , where  $\alpha = 95\%$  represents the coverage probability, see equation (5). Figure 4 illustrates the chart obtained by plotting the values of J integral versus the measured values of crack growth, which gives the J-R curve of the material.

$$U = \frac{Z_{97,5\%} - Z_{2,5\%}}{2} \quad (5)$$



**Figure 4.** J-R curve obtained.

Table 2 presents the obtained results of calculation of J-integral values and the respective expanded uncertainty for each specimen analyzed. The results were analyzed along the curve aiming finding some discrepancy. The obtained data illustrates the growth of the expanded uncertainty when the specimen's fracture toughness is greater and the similar values obtained when the results are closer, see specimens 1 and 3.

**Table 2.** Results of J-integral values and expanded uncertainty.

Specimen	1	2	3	4	5	6	7
<b>J-integral [N/mm]</b>	5.97	8.07	5.88	9.59	14.36	12.98	19.12
<b>Expanded uncertainty [N/mm]</b>	0.07	0.09	0.07	0.11	0.16	0.14	0.20

#### 4. CONCLUSIONS

The measurement uncertainty is the instrument used to verify the quality of a measurement system. Considering this factor gives an important tool to predict the applicability of the measurement systems in use, as for test methods. The Monte Carlo method has become a way to avoid problems caused by the particularities of the GUM uncertainty framework. J-integral, with its non-linear model, had the Monte Carlo method adapted to this particular case and the results showed that the method applied presented consistent values. Besides the small quantity of specimens tested, the similarity of the uncertainty, mainly between the specimens tested in closer areas of the J-R curve, illustrates that the measurement method applied offers a solution for the obtainment of J integral results [7].

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